

Sudden and Gradual Processes of Insight Problem Solving: Investigation by Combination of Experiments and Simulations

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Abstract

The insight process is generally characterized by suddenly finding a solution in problem solving. On the other hand, recent cognitive studies have indicated that the insight process involves a gradual process of approaching the solution. In this study, we investigated such bifacial characteristics of the insight process from the viewpoint of a hypothesis search process by psychological experiments, and tried to explain how these characteristics arise using a computer simulation model. In the computer simulation model, we assumed that the insight process consists of two qualitatively different types of hypothesis search processes in which reinforcement and chunking learning methods are used. The results of computer simulation models indicated that both sudden and gradual characteristics arose from the interaction of these processes in problem solving.

Introduction

Bifacial Characteristics of the Insight Process

Insight problem solving is different from normal problem solving in many aspects. In normal problem solving, we approach to a solution through incremental steps. On the other hand, in insight problem solving, we meet an impasse because we use experiences of past problem solving as negative factors and then suddenly find a solution (Metcalf and Wiebe, 1987; Smith, 1995). In contrast to the sudden attainment to a solution, the process of insight problem solving develops through gradual steps. In this context, the insight process is described from the viewpoint of mental constraint relaxation. First, mental constraints arise from past experiences and the structures of problems, and we meet an impasse because these constraints prevent us from reaching a solution. Gradually these mental constraints are relaxed. Therefore a search that does not follow these mental constraints gradually increases, and we reach a solution (Knoblich et al., 2001; Suzuki et al., 2003).

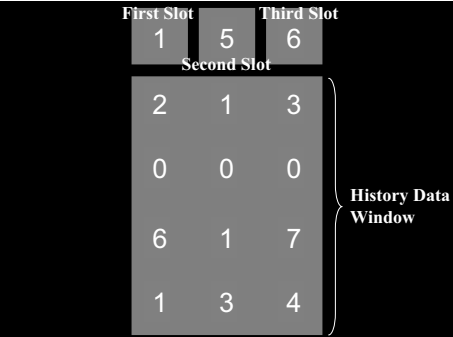
Purpose

How can we systematically explain such bifacial characteristics of the insight process? The purpose of this study is to construct a computational model for the insight process from a viewpoint where it consists of both sudden and gradual processes. In this study, we attempt to understand the insight process by conducting psychological experiments and using computer simulations.

Insight Task

Overview

In this study, we proposed and used a discovery task that asked subjects to find a rule for predicting a digit. An example screenshot of the task is shown in Figure 1. The display consists of three slots, and in each slot a single digit rotates at a speed that prevents the subjects from perceiving each digit. A history data window indicates the instances of the past four trials. The digit in the third slot is controlled by an unknown rule (*target rule*). If the subjects find the target rule, then they can predict the digit in the third slot. The mission of the subjects is to find the target rule and predict the digits in the third slot.



First Slot	Second Slot	Third Slot
1	5	6
2	1	3
0	0	0
6	1	7
1	3	4

Figure 1: Screenshot of discovery task used in this study.

Subjects are required to predict the digit in the third slot after the two digits in the first and second slots stop rotating. A series of the procedure, stopping the first and second slots and then predicting and confirming the third digit, is called a *trial*. A history data window below the three slots shows the results of the past four trials as history data. The rules reported in each trial by the subjects are called hypotheses, which are proposed in the process of hypothesis formation and testing.

This is a discovery task that requires insight. Therefore, the task is manipulated to lead the subjects to find a sham rule called a *blocking hypothesis*, which differs from the target rule. The target rule: “the third digit in the n -th trial is determined by adding three to the third digit in the $n-1$ th (previous) trial” with a vertical relation. The blocking hypothesis: “the third digit is equal to the sum of the first and second digits” with a horizontal relation.

Subjects are required to predict the digit in the third slot after the two digits in the first and second slots stop rotating. In the initial eight trials, by controlling the first and second

digits, the third digit is consistent with the sum of the first and second digits while maintaining the target rule existing across the vertical row (see the example display in Figure 1; this rule is confirmed as follows: $1 + 3 = 4$, $6 + 1 = 7$, $0 + 0 = 0$, $2 + 1 = 3$, $1 + 5 = 6$ in each column, and a series of 4, 7, 0, 3, 6 from bottom to top in the third row). Therefore, the subjects are guided to the blocking hypothesis with the horizontal relation as a sham rule. After the first eight trials, a digit predicted by the blocking hypothesis gradually disagrees with an actual third digit. Consequently, from the ninth trial subjects begin to receive negative instances called *negative feedback* that disconfirm the blocking hypothesis.

Definition of Types of Hypothesis Spaces

The process through which subjects find the target is considered as a process of searching for a hypothesis space. A group of hypotheses that have a common regularity constitutes a hypothesis space; therefore the target rule and the blocking hypothesis belong to different problem spaces. In this study, the relation between hypotheses and hypothesis spaces is defined as follows.

Blocking Hypothesis Space A set of hypotheses characterized by a horizontal relation is defined as a blocking hypothesis space.

Blocking Hypothesis: as described above.

Horizontal Hypotheses: rules characterized by a horizontal relation other than the blocking hypothesis.

Target Space A set of hypotheses characterized by a vertical relation is defined as a target space.

Target Rule: as described above.

Vertical Hypotheses: rules characterized by a vertical relation other than the target rule.

In addition to these hypotheses, subjects reported hypotheses characterized by both horizontal and vertical relations, which are involved neither in the blocking hypothesis space nor in the target space, such as the same digits arranged diagonally.

Psychological Experiments

In psychological experiments, we requested subjects to solve the discovery task as described above and analyzed the process of the subjects stumbling into insight.

Proposed Hypothesis and Hypothesis Search

The subjects repeated the following procedures to find the target rule. First, they predicted the digit in the third slot by proposing a hypothesis (we call this process *hypothesis proposal*), and second they stopped the third slot to confirm this hypothesis. In this study, we captured *hypothesis proposals* by using subjects' verbal reports.

However, verbalized hypotheses do not indicate all hypotheses searched by the subjects. A *hypothesis search* phase of searching for a huge variety of hypotheses, that is, a phase of seeking various possibilities, would probably exist until a hypothesis is proposed by the *hypothesis proposal* phase. In this study, such non-verbalized *hypothesis search* in solving the task became apparent by capturing subject eye movements. For example, searching for the *blocking hypothesis*

is identified from horizontal eye movements and the *target space* from vertical eye movements.

Method

Subjects Twenty-four undergraduate students participated in this experiment.

Procedure The subjects start a trial, report a predicted rule as a hypothesis after the first and second slots stopped, and discontinue the third slot to confirm the hypothesis. Such hypothesis proposing and testing as a trial lasted for a maximum of 55 minutes until finding the target rule.

Results

We excluded eight of the twenty-four subjects because they could not form a blocking hypothesis through the initial eight trials or could not provide any fine eye movement data. In this paper, we are only concerned with the results of the five successful subjects who found the target rule (for details see Terai and Miwa (2003)).

Transition of Proposed Hypotheses Figure 2 shows the transition of the proposed hypotheses in successful subjects. The horizontal axis indicates the number of trials, and the vertical axis indicates each type of hypothesis and the hypothesis space described above.

Figure 2 shows that all successful subjects found the blocking hypothesis by the time they reached the ninth trial. After the ninth trial, the subjects began to receive negative instances for the blocking hypothesis, proposing other hypotheses than the blocking hypothesis. However, hypotheses that existed outside of the *blocking hypothesis space* were almost never proposed, confirming that the subjects continued to search for the blocking hypothesis space. This result indicates that the subjects had encountered an impasse. Figure 2 shows that the discovery of the target rule seemed to occur suddenly from the state where subjects were searching for the blocking hypothesis space, rather than by a gradual shifting through potential hypothesis spaces.

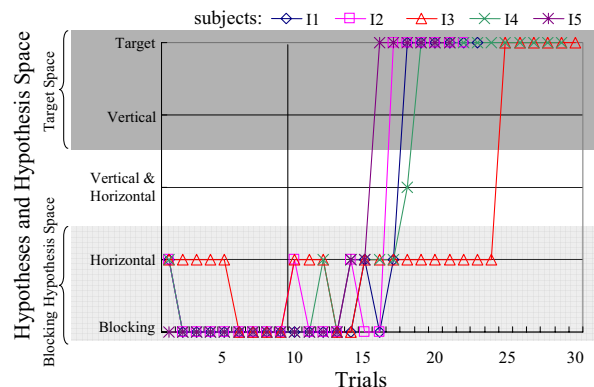


Figure 2: Transition of proposed hypotheses in successful groups.

Transition of Hypothesis Search Next, we analyze the process of subjects searching for hypothesis spaces leading to finding the target rule by using their eye movement data, which were obtained as transition patterns of the fixation of eye movement.

The transition of the horizontal and vertical eye movements of successful subjects is shown in Figure 3. The horizontal axis shows the number of trials: the first nine trials, three trials after negative feedback was given, and four trials before and after the target rule was discovered. The vertical axis indicates the ratio of each type of eye movement to all types of eye movement (horizontal, vertical, diagonal, and fixed).

Figure 3 shows that the horizontal eye movement, corresponding to search for the blocking hypothesis space, dominated until the subjects reached the ninth trial. By contrast, after they were given negative feedback, the ratio of horizontal eye movement gradually decreased whereas the ratio of vertical eye movement gradually increased. This result indicates that the search for the blocking hypothesis space gradually decreased and the search for the target space gradually increased.

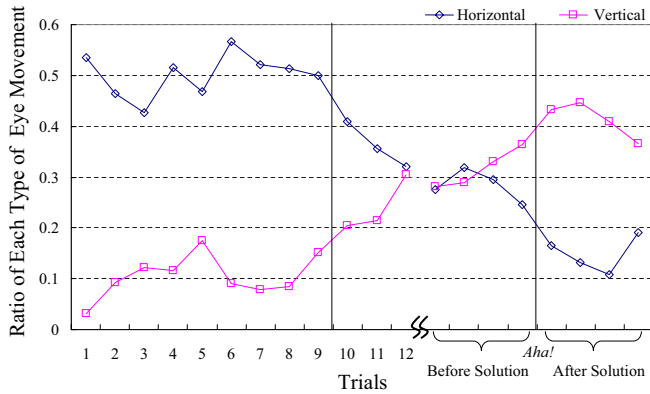


Figure 3: Transition of hypothesis search in successful groups.

Summary

In psychological experiments, we captured the transitions of *hypothesis search* and *hypothesis proposal* in the process of hypothesis formation and testing by using subjects' verbal reports and eye movement analysis. From the viewpoint of the former, regarding their hypotheses as *hypothesis proposal*, the discovery of the target rule seemed to occur suddenly from falling into an impasse. On the other hand, the analysis of subject eye movement as *hypothesis search* revealed that searching hypotheses gradually varied with the development of problem solving after negative feedback was given.

Computer Simulation Model

Process of Insight Problem Solving as a Hybrid Model

The suddenness and gradualness of insight problem solving are confirmed through psychological experiments in this study. In this section, we explain how such bifacial characteristics of the insight process arise by using a computer simulation.

In our model we assumed that the insight process consists of two different processes. One tries to form hypotheses by exploiting past experiences. This process is called the *knowledge driven process* in this model. Another tries to form hypotheses according to feedback from the environment. This process is called the *adaptive process* in this model. In this

study, we will explain the bifacial characteristics of the insight process as a phenomenon arising from interaction between the *adaptive process* and the *knowledge driven process*.

Outline of the Model

Attributes of a Hypothesis A hypothesis formed in the task used in the psychological experiments consists of two attributes: *sequence of data* and *numerical relation*. Consider rule “slot 3 = slot 1 + slot 2” as an example. *Sequence of data* indicates the positional relation of the associated digits in Figure 1. In this case, *sequence of data* horizontally indicates “slot 1, slot 2, slot 3.” This attribute corresponds to a manner of search: how subjects search for the experimental stimulus of the task and obtain data from it.

On the other hand, *numerical relation* corresponds to a numerical rule existing among data involved in the *sequence of data*. In this case, the *numerical relation* is “addition.” This attribute corresponds to numerical knowledge retrieved from the long-term memory of the subjects.

In this study, we captured the insight process by focusing on the searching hypothesis spaces. The attribute that corresponds to search for hypothesis space is *sequence of data*; therefore in the following, we focus only on *sequence of data*.

Adaptive Process and Knowledge Driven Process The learning of *sequence of data* is performed through confirmation or disconfirmation of the formed hypotheses. In the model, hypothesis formation is conducted by assuming two different processes: adaptive and knowledge driven.

Adaptive Process The *adaptive process* performs hypothesis formation using reinforcement learning. Patterns of *sequence of data* that constitute hypotheses are learned as obtaining data behaviors in reinforcement learning. To be more precise, the model forms hypotheses based on both the *sequence of data* obtained from reinforcement learning and the *numerical relation* retrieved from the long-term memory that satisfy *sequence of data*. In the *adaptive process*, obtaining data behavior from the stimulus of the task changes based on the experiences of confirmation or disconfirmation of the hypotheses.

Knowledge Driven Process In the *adaptive process*, a generated hypothesis is gradually adopted based on the success or failure of forming hypotheses. On the other hand, in the *knowledge driven process*, past experiences are exploited as *chunks* to form hypotheses. *Chunks* are successful instances in the *adaptive process*: in this case, particular *sequence of data* patterns. For example, when a hypothesis is formed using data acquired in a sequence of “slot 1, slot 2, slot 3” and confirmed repeatedly in the *adaptive process*, this *sequence of data* is extracted as a specific pattern, i.e., a *chunk*, that will be exploited in the *knowledge driven process*.

In the *adaptive process*, when such data acquisition patterns become ineffective in hypothesis formation, the patterns cannot be kept by relearning because learning the obtaining data behavior is probabilistic. By contrast, *chunks* correspond to specific patterns of data acquisition, such as obtaining data from horizontal or vertical directions, and so on. Thus, once a data acquisition pattern is learned as a *chunk*, then this pattern is maintained even if it becomes ineffective in hypothesis formation.

Interaction Between the Adaptive Process and the Knowledge Driven Process The *adaptive process* and the *knowledge driven process* interact with each other in interactions that consist of bottom-up and top-down learning. The former corresponds to extracting *chunks* that develop from the *adaptive process* to the *knowledge driven process*. Top-down learning corresponds to adjusting the parameters of reinforcement learning that develop from the *knowledge driven process* to the *adaptive process*.

Hypothesis Search and Hypothesis Proposal The rule discovery process consists of *hypothesis search* and *hypothesis proposal*. *Hypothesis search* is carried out through the *adaptive process* and the *knowledge driven process* as described above. The model searches for hypotheses while alternating between the two processes that occur in a certain probability. When an appropriate hypothesis is discovered in *hypothesis search*, *hypothesis proposal*, which follows *hypothesis search*, proposes this hypothesis as output, and the model moves to the next trial after discontinuing the third slot. On the other hand, if an appropriate hypothesis is not discovered in *hypothesis search*, then *knowledge driven process* proposes a hypothesis as output that was formed using a *chunk*.

Computer Simulations

We used three alternative models and two different tasks to estimate the validity of these models in computer simulations.

Models The hybrid model is as described above. As an insight model we compared it with both the adaptive and random models to indicate the validity of the hybrid model. The adaptive model is a model in which the *knowledge driven process* in the hybrid model is replaced by the *adaptive process*; in other words, this model forms a hypothesis by only using reinforcement learning. In the random model, both the *knowledge driven process* and the *adaptive process* in the hybrid model are eliminated; data acquisition and hypothesis formation are performed randomly.

Tasks We used two tasks, insight and non-insight, to estimate the three models above and to indicate the validity of the hybrid model as an insight model.

The insight task was identical to the task used in the above psychological experiments. On the other hand, the non-insight task was constructed to allow the blocking hypothesis and the target rule to exist in the same hypothesis space. The blocking hypothesis in the non-insight task is identical to that in the insight task. The target rule is, “the third digit is consistent with the sum of the units’ and tens’ digits of the addition of the first and second slots.” For example, when the first and second slots are nine and eight, the third slot becomes eight as an addition of seven (i.e., the units’ digit) and one (i.e., the tens’ digit).

Results

Performances of Models Figure 4 shows the ratio of discovering the target rule by the fortieth trial where each model solves two types of experimental tasks. Each ratio is the average of discovering the target rule, calculated through one thousand simulations. The performances of the random model were the same when solving either the insight task or

the non-insight task. This happened because the difference in the structures of the two tasks did not influence the performance of discovering the target rule since the hypothesis search was performed randomly in the random model. Thus, the random model can be considered a model in the control condition.

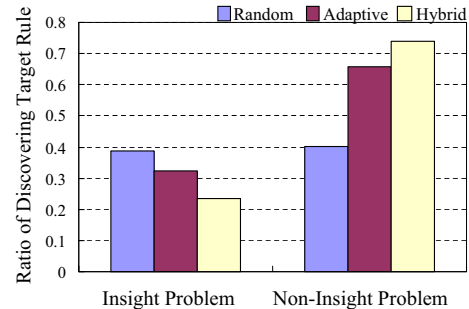


Figure 4: Models’ performances of discovering the target rule.

Insight Task Figure 4 shows that when solving the insight task, the performance of the random model is the highest; second is the adaptive model, and the worst is the hybrid model. In the adaptive model, performance apparently decreases more than in the random model because, when this model found the blocking hypothesis in the initial eight trials, an invalid *sequence of data* constituting this hypothesis was reinforced. Furthermore, in the hybrid model, performance also apparently further decreases because a *chunk*, blocking the discovery of the target, was formed due to finding a blocking hypothesis.

Non-insight Task By contrast, when solving the non-insight task this pattern is reversed. For the non-insight task, the performance of the hybrid model is the highest; second is the adaptive model, and the worst is the random model. In the non-insight task, apparently the experience of discovering the blocking hypothesis by the eighth trial, where the target rule exists in the same problem space as the blocking hypothesis space, which facilitated the discovery of the target rule; in the hybrid model, a *chunk* was learned from this experience that worked effectively. Although learning also occurs in the adaptive model, its performance is lower than the hybrid model because relearning occurs whenever hypotheses are disconfirmed, increasing the search for other hypothesis spaces more than the blocking hypothesis space. The performance of the random model was the lowest because it does not learn.

Hypothesis Search and Hypothesis Proposal It is confirmed that the performance of the hybrid model for the insight task decreased as well as in the above psychological experiments. Next, we focus on the process of searching for hypothesis spaces and proposing hypotheses in each model that solves the insight task.

Figure 5 shows the transition of the hypothesis search and the proposed hypotheses in each model solving for the insight task. The transition of the hypothesis search indicates a change in the formed hypothesis in the *hypothesis search* phase. This data correspond to the psychological data captured using eye movement analysis in the psychological experiments. The transition of proposed hypotheses indicates

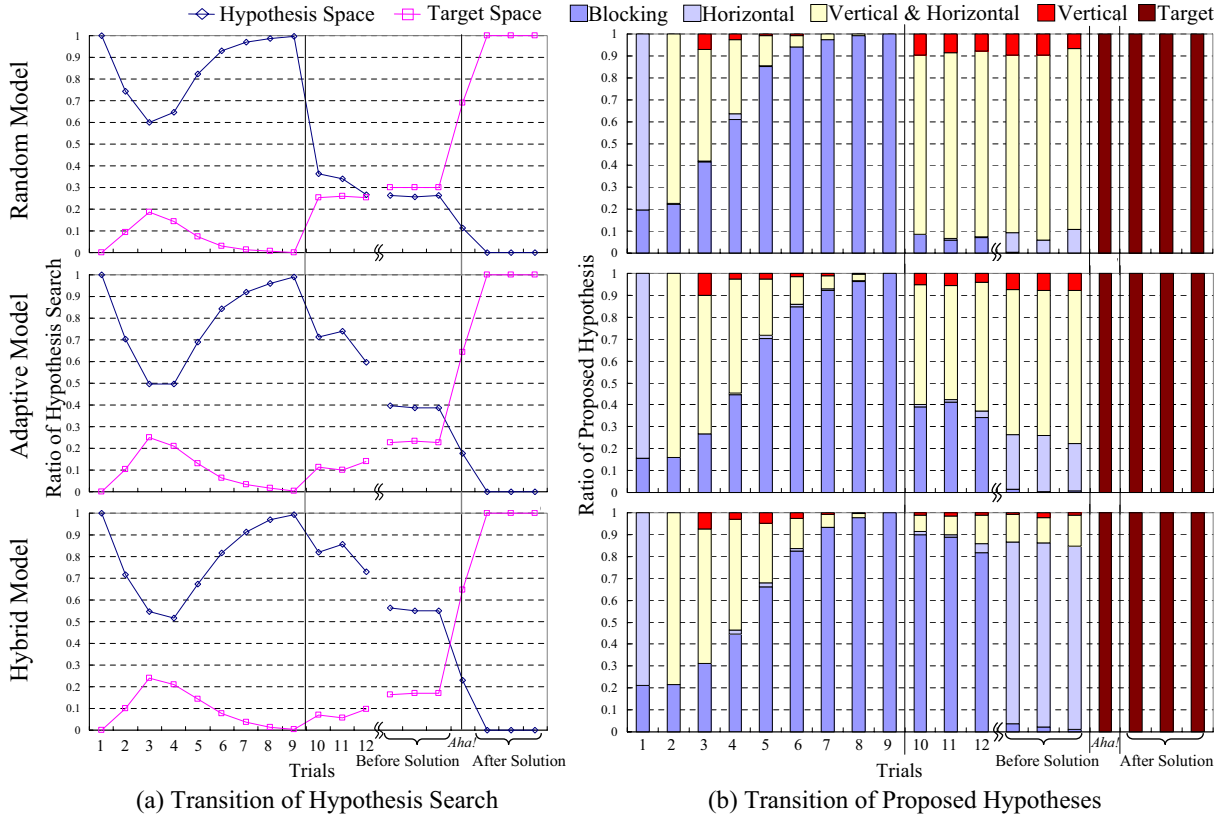


Figure 5: Transition of Hypothesis Search and Proposed Hypotheses.

the change of an output hypothesis in the *hypothesis proposal* phase. This data correspond to the psychological data captured by subject verbal reports in psychological experiments.

In the results of the transition of the hypothesis search (Figure 5 (a)), the horizontal axis shows the number of trials that indicate the first nine trials, the following three trials after negative feedback was given, and the three trials before and after the target rule was discovered. The vertical axis indicates the ratio of the number of hypotheses searched for in each hypothesis space in the *hypothesis search* process to the number of hypotheses searched for in all hypothesis spaces. In the results of the transition of the proposed hypotheses (Figure 5 (b)), the horizontal axis uses the same measurement as Figure 5(a); the vertical axis indicates the ratio of the number of hypotheses proposed in the *hypothesis proposal* phase to the number of all proposed hypotheses.

In the *hypothesis proposal*, it is confirmed that each model found the blocking hypothesis through the initial eight trials. Next we will discuss the behaviors of each model after negative feedback was given.

Random Model After negative feedback was given, searching for the blocking hypothesis space drastically decreased, and the model moved to a random search. Correspondingly, in the *hypothesis proposal*, the random model proposed a wide variety of hypotheses without fixating on a search for the blocking hypothesis space.

Adaptive Model After negative feedback was given, searching for the blocking hypothesis space gradually decreased. Correspondingly, in the *hypothesis proposal*, other hypotheses than those in the blocking hypothesis space increased. The transition pattern of *hypothesis search* is almost

identical to the *hypothesis proposal* in the adaptive model. This similarity was also observed in the behavior of the random model.

Hybrid Model On the other hand, the hybrid model showed a qualitatively different pattern from those shown by other models. In the *hypothesis search*, after negative feedback was given, searching for the blocking hypothesis space gradually decreased. As for the *hypothesis proposal*, hypotheses were proposed while being fixated from the blocking hypothesis space even after negative feedback was given, confirming that the discovery of the target rule occurred suddenly. This means that the *hypothesis proposal* pattern was relatively different than the *hypothesis search* pattern.

Summary

The bifacial characteristics of insight process, gradualness in *hypothesis search* and suddenness in *hypothesis proposal*, were confirmed by the hybrid model through computer simulations. These results imply that such bifacial characteristics of the insight process arise from the interaction between the *adaptive process* and the *knowledge driven process*.

Discussion

In this section, we discuss the elements that characterize insight: fixedness and suddenness/gradualness in the insight process.

Fixedness

In psychological experiments, after negative feedback was given, although searching for hypothesis spaces gradually

varied, hypotheses were fixedly proposed from the blocking hypothesis space. Such fixedness is one of the representative phenomena characterizing insight, and this leads to impasses.

Fixedness is also generally found in daily life. Referred to as functional fixedness, problem solving is inhibited by the constraints of the daily usage of objects, even though we are not compelled to do so (Duncker, 1945). Moreover, such fixedness occurs not only in functional aspects but also in the strategy selection of problem solving and memory retrieval (Luchins and Luchins, 1950; Smith and Blankenship, 1991).

This phenomenon was also confirmed in the computer simulation results of this study. When solving the insight task, if the blocking hypothesis existing in a different hypothesis space than the target space was given, the hybrid model produced fixedness on searching for the blocking hypothesis space. Moreover, the results clearly showed that the performance of the hybrid model was the lowest in solving the insight task because *chunks* formed by bottom-up learning lead the model to search for blocking hypothesis spaces.

The simulation results suggest that fixation arises from the accumulation of past experiences as chunks. However, in many cases such a human thought process facilitates problem solving for example non-insight problems, where such a nature effectively promotes the discovery of target rules. In short, in non-insight problems, searching for hypotheses in the target space is facilitated by chunks formed from past experiences. When the non-insight task was given to each model in the simulations, the performance of the hybrid model was the highest.

Actually previous research has pointed out two aspects of chunks: while their existence for utilizing past experiences effectively facilitates problem solving in non-insight problems, they also generate fixedness during insight problems. Our research has provided explanations for the positive and negative aspects of chunks using actual functioning computational models.

Suddenness/Gradualness

In this study, psychological experiments verified that the bifacial characteristics of the insight process exist in the gradualness of the *hypothesis search* and in the suddenness of the *hypothesis proposal*. Such bifacial characteristics of insight appeared in the models' behavior as follows.

In the hybrid model as an insight model, the search for hypotheses is performed by alternately switching between the *adaptive process* and the *knowledge driven process*. When an appropriate rule is found in the *hypothesis search* phase, this rule is proposed as a hypothesis. If an appropriate rule is not found in this phase, then a hypothesis is proposed by the *knowledge driven process* based on past successful experiences. Hypothesis formation by the *knowledge driven process* is performed by using *chunks* as past successful experiences.

In other words, this hypothesis formation is constrained by *chunks*. Such an insight process is characterized by both a *strong constraint* and a *weak constraint* from the viewpoint of mental constraint relaxation.

Strong Constraint: This constraint appears in hypothesis formation based on *chunks* of past experiences in the *knowledge driven process*.

Weak Constraint: This constraint appears in hypothesis formation based on reinforcement learning in the *adaptive process*.

These two qualitatively different constraints are considered critical for adapting to environments in parallel while exploiting past experiences. Our computer simulations demonstrated that suddenness and gradualness in the insight process arise from these two constraints.

Conclusion

In this study, we discussed the insight process from falling into an impasse to sudden discovery in both psychological experiments and computer simulations. Proposed hypotheses by subjects in psychological experiments showed sudden changes at the moment of discovering the target rule. By contrast, searching for hypotheses captured by eye movement analysis showed a gradual transition after negative feedback was given. Even if in the final selection phase captured by verbal reports, representation suddenly changes with flash of luminance at the moment of finding the solution; in the process of achieving insight, blocking constraints are gradually relaxed after receiving feedback from the environment. In this study, it was confirmed that the bifacial characteristics of the insight process arise from the interaction between the *adaptive process* guided by a weak constraint and the *knowledge driven process* by a strong constraint.

Acknowledgment

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