Study on Support of Learning from Examples in Problem Posing as a Production Task

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Abstract: Problem posing is identified as an important activity in mathematics education as well as problem solving is. While problem solving is a comprehension task, problem posing is regarded as a production task because it requires diverse thinking and generation of novel ideas in some ways. In problem posing, it is important but difficult for learners to generate diverse problems. In this study, we propose a strategy for learning from examples in problem posing in order to promote diverse problem posing by learners. We introduce an activity of imitation that is widely adopted in domains of creative generation tasks. We also implement a system that supports learning by imitation in a task of posing mathematical word problems. Our system presents a learner with cases of problems and their generation processes, and it then has the learner engage in reproducing cases by following the processes.

Keywords: Learning from examples, imitation, problem posing, production task

Introduction

In general mathematical learning, students solve many problems provided by a teacher or textbooks. But, besides problem solving, problem posing has also been identified as an important activity in mathematics education. In fact, some mathematicians and mathematics educators have pointed out that problem posing lies at the heart of mathematical activity [2, 11, 12]. Problem solving and problem posing are not entirely different cognitive activities but are closely related (e.g., [1, 13]). On the other hand, they are of course different in features and formats of their tasks. Problem solving is a convergent task, in which a learner extracts a mathematical structure from given information and reaches a correct answer. In contrast, problem posing is a synthetic activity and a divergent task that fundamentally has multiple answers. Here, we call the former task as a comprehension task, and the latter as a production task. Problem posing requires productive thinking. In problem posing, a learner has to generate new ideas in some way because new problems cannot be generally composed only from given information in the task.

Learning from examples is necessarily adopted in initial skill acquisition. Regardless of problem domains and task formats, the examples (e.g., example problems in mathematics or physics, papers of related work in academic research, existing works in art or mechanics design, and performances by experts in music or sports) are indispensable for learning. However, regarding how to learn from such examples and how to support the learning, we have to design specific learning methods or environments depending on problem domains and task formats.
This study discusses and proposes an approach to support learning from examples in the domain of problem posing as a production task. We then implement a support system for the learning. Here, our goal in the learning support is to facilitate diverse problem posing of learners by generating and combining various ideas.

1. Support for Learning from Examples in Problem Posing

1.1 Significances and Difficulties of Problem Posing

Many benefits are gained from problem posing, such as enhancing problem solving ability and grasp of mathematical concepts, generating diverse and flexible thinking, alerting both teachers and students to misunderstandings, and improving students' attitudes and confidence in mathematics [2, 12]. Although problem posing is hardly adopted in general education due to some constraints in practical classrooms, it is a critical skill as well as problem solving.

In problem posing, we can design various task formats by specifying constraints or materials given to learners. In general, problem posing is a task to generate new problems by appropriately combining contextual settings expressed in problem texts (situations) and mathematical structures of solutions (solutions). In learning by problem posing, it is not useful for learners to repeatedly generate similar problems. It is crucial to generate diverse problems by extracting several solutions from one situation or by recalling multiple situations to which one solution can be adapted. However, such diverse problem posing is considerably difficult for learners. It has been confirmed that problems generated by novice learners lack diversity [3, 10].

It is frequently argued that there is a relationship between problem posing and creativity [9, 12], because problem posing needs generation of new ideas. As mentioned above, problem posing is a productive task so that it is expected to train learners' diverse thinking for idea generation. To gain the benefit of diverse thinking, however, we have to make learners generate diverse problems. Therefore, we have to propose a supporting method to promote diverse problem posing by learners. That aims to improve performance of problem posing by learners through facilitation of the learners' diverse thinking that is an ability associated with creativity.

1.2 A Supporting Method in Previous Study

We have approached to facilitate diverse problem posing by presenting examples of problems to learners in our previous study [7]. We adopted a task format where a learner generated new problems in the domain of a given example problem. Posed problems by the learner were evaluated using categories that were specified by similarities in situations and solutions between the example problem and posed problems. Figure 1 indicates the categories. Learners must pose a variety of problems in multiple categories by altering a situation and/or solution of the example problem, though, their posed problems tend to converge on Category-I / I without any support.

In the previous study, we have implemented a system that presents learners with a variety of cases as examples of output in the problem posing task and helps them in comparing the cases with the example problem or their posed problems. Experimental evaluations of the system confirmed that it can facilitate learners’ diverse problem posing to some extent. Posed problems in categories other than I / I increased when the learners had been shown cases different from the example problem in situations or solutions. However, the extent of the posed problems was still limited even with the support by the system. We have
conducted a further investigation where learners were asked to evaluate cases presented by
the system [8]. The results of the investigation revealed that there were gaps between
comprehension of the cases and production of problems by the learners. They didn’t
necessarily adopt ideas used in generation of a case for their problem posing, even if they
evaluated the case as a good problem.

Although the system presents cases to learners and prompts them to compare the cases
with their posed problems, it doesn’t give any instructions about how to learn from the
cases. The cases are merely shown to the learners. We haven’t examined how the learners
learned from the cases. The learners may have simply read the presented cases, or they
may have solved the cases. In other words, the learners may have understood the cases
through doing a comprehension task. If a learner examines a case through a production
task, s/he may understand more about its background (e.g., operations adapted in altering a
solution). According to this insight, we propose a method for learning from examples
through a production task.

![Figure 1. Categories for evaluating posed problems](image)

1.3 Learning from Examples through a Production Task

We propose a method of learning from examples through a production task by introducing
a learning activity adopted in productive task domains. One of the typical learning
activities in such domains is imitation of examples. The imitation, in which a learner
reproduces an existing example work, has been long adopted as a major learning activity
in the domain of creative generation, such as art. Relation between creation and imitation
has been usually argued in such creative domains. Ishibashi and Okada [5] documented
that imitating examples can prompt imitators' understanding of the examples and their
conceptual background, and experimentally confirmed that imitation facilitates creative
performance by imitators. In their experiment, subjects were engaged in an artistic
drawing task and were asked to create copies of a presented example. They found that the
subjects deeply understood the example through imitation of it. They then argued that
understanding of the example elicited understanding of representations of the subjects
themselves.

Based on the findings by Ishibashi and Okada, we adapt imitation to the learning from
examples in problem posing. Imitation here is to reproduce a problem identical to a
presented case. The learning by imitation aims to have the learner understand background
knowledge and ideas to formulate the case from the viewpoint of the creator. However, if
a learner is asked to reproduce a presented case, s/he would merely duplicate characters
and symbols composing a problem text and a solution of the case. The learner would learn
nothing from such duplication. Thus, we propose a supporting method of learning by
imitation. In the learning, a learner is given information of processes where a problem as a
case is generated. The learner then constructs the problem by following the generation
process information. We implement a system that supports the learning by imitation.
2. Implementation of a Support System

Our system gives a learner an example problem and the problem posing task identical to the previous study [7] described in Section 1.2. Support by our system aims to facilitate the learner’s diverse problem posing through learning by imitation of cases. In our system, the learner learns concrete ideas and methods to compose new problems from the imitation. That is expected to expand the variety of problems the learner can pose. Same as the previous study, we select a problem domain of word problems solved by simultaneous equations.

Our system can present problems as cases generated by altering the example problem and information of processes where the cases are generated from the example problem. A learner reproduces problems identical to the presented cases by following the generation process information. Hirashima and his colleagues developed a learning environment similar to our system [4]. In their environment, a learner poses new problems by changing a given problem. In both of their environment and our system, learners learn how to pose new problems from another problem. However, the study by Hirashima et al. intends to improve learners’ understanding of solutions through the problem changing. In contrast, our purpose is to diversify learners’ problem posing.

In implementation of our new system, we incorporate functions of an automatic generation system of mathematical word problems [6] and the support system for problem posing [7] mentioned in the previous section. The generation system semi-automatically generates new problems and constructs a database that has various problems in situations and solutions.

2.1 Provision of Generation Process Information

As described above, our system presents cases and process information about how the cases are generated. The generation system provides those data.

![Figure 2. Basic concept of the problem generation](image-url)

The generation system uses problems initially stored in problem generation. Figure 2 indicates basic concept of the problem generation. The system forms an episode comprising a base example problem (base) and a new analogical instance (new instance) which can be generated by altering the base. A new output problem (output) is generated from an input problem (input) by mapping relationships in the episode. In Figure 2, new
problems B-2 and B-3 are generated from A-1, A-2, A-3, and B-2, which are initially stored in the system. After single problem generation, the system simultaneously generates output and log data recording how the output is generated from input. Our new system uses those data. It uses problems generated from one identical to the example problem used in the problem posing task as cases. Moreover, it creates generation process information that is presented to learners from log data recorded in the generation system.

2.2 Architecture of our System

Figure 3 indicates the architecture of our system implemented in the current study. Our system is comprised of three interfaces, a problem input interface to receive problems from learners, a process display interface to show generation process information of cases, and a feedback interface to show evaluations of problems posed by learners and cases presented to the learners. Situation estimation models and an equation parser are offered from the support system in the previous study, which are used to understand situations and solutions of problems input by learners. A dictionary database used to understand keywords in problems, a problem database to provide problems as cases, and a log database are offered from the generation system. Generation process information presented to learners is created from log data in the log database by a log transformation tool.

2.3 Procedures of Learning with our System

In learning with our system, a learner is first given an example problem. In a learning-by-posing mode, the learner is prompted to generate a new problem from it. The learner inputs a posed problem into the problem-input interface. Our system extracts a situation and solution of the posed problem and then gives a feedback to the learner. Figure 4 shows a screenshot of the feedback interface. The feedback interface indicates evaluation of the posed problem using the same representation as in Figure 1 (middle in Figure 4). Simultaneously, it retrieves and presents some cases (lower in the figure). The example problem is shown in the upper in the figure. Those procedures in the learning-by-posing mode are same as the support system in the previous study.

Figure 4. Screenshot of feedback interface

Figure 5. Screenshot of problem input interface and process display interface
A learner is prompted to reproduce a presented case under a *learning-by-imitation mode*. Figure 5 shows a screenshot in the mode. The right side of the figure is the problem input interface, and left side is the process display interface indicating how the case is generated from the example problem. The presented case itself is hidden while the learner is working with the problem input interface in the learning-by-imitation mode. Information needed in forming the case is completely provided by the process display interface. The learner reproduces the case by composing the provided information. Through this activity, the learner is expected to understand ideas used in posing of the presented case from the viewpoint of a problem poser.

### 3. Preliminary Evaluation

We conducted a preliminary evaluation to verify whether our system can expand the variety of learners’ problem posing. Seven undergraduates participated in the evaluation as learners. Procedures in the evaluation are as follows.

1. **Pre test**
   
The participants were presented an example problem in a domain of word problems solved by unitary equations and were then asked to generate two problems in the domain.

2. **Learning with our system**
   
The participants were presented two cases and reproduced them according to their generation process information provided by our system. One of the cases was randomly selected from problems in category-D / I, and the other was in category-I / D. Prior to the reproduction, they had practiced operation of our system.

3. **Post test**
   
The participants were asked to generate two more problems from the example problem in the pre test. They had been instructed to pose problems different from those in the pre test.

![Figure 6](image.png)

*Figure 6. Proportions of participants who only generated problems in category-I / I*

Figure 6 indicates the proportions of participants who generated only problems in category-I / I in pre and post tests. As the figure indicates, more than half of the participants generated only problems in category-I / I in the pre-tests. In the post-test, the number of such participants disappeared. This result indicates that problem posing by the participants was diversified. That is basically same to the results obtained in evaluations of the support system in previous study [7]. We haven't found the additional effect by
imitating cases in this evaluation. It is our important work in next step to verify the effect of learning through imitation in problem posing.

4. Conclusions

In this study, we discussed an approach to support learning from examples in problem posing as a production task. We proposed a method of learning by imitation where a learner reproduces cases by following generation process information of the cases. We also implemented a system that supports the learning by imitation. As mentioned above, one of our important future work is to evaluate the learning effect of our system. Furthermore, we have to study differences between learning effects by a production task and by a comprehension task through experimental evaluations of our system.

References